Comparative studies of numerical field calculation methods for the optimization of high voltage equipment

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I. Introduction

The paper will outline modern techniques for numerical field calculation with a detailed study of advantages and disadvantages of methods like Finite Element Method (FEM), Charge Simulation Method (CSM) and Boundary Element Method (BEM). All methods presented in the paper can be used for the calculation of electrostatic fields in order to improve high voltage components. This equipment is becoming increasingly important due to the changes in Germany's energy supply system.

By applying a basic but yet relevant example, it will be demonstrated which methods are most efficient for the calculation and optimization of electric fields in high voltage equipment.

Furthermore, new techniques for optimization of three dimensional fields and components will be presented using appropriate mathematical optimization algorithms along with massive parallel computing technology.

II. Calculation of electrostatic fields

The knowledge of the electric field is the basis of the design of high voltage systems. Although insulation properties and environmental conditions are an important aspect, the electric field strength is the decisive parameter for the appearance of electric discharges. High voltage equipment is stressed by AC fields with frequencies of 50, 60 or 16 $\frac{2}{3}$ Hz or by DC fields. The wavelength of these frequencies is very large in comparison to the dimension of the HV elements. Therefore the field can in first approximation be viewed as quasi-stationary, and methods for calculating electrostatic fields can be applied [1], [2].

The aim of field calculation is to determine the potential $\Phi(x, y, z)$ and the electric field strength

$$\vec{E}(x, y, z) = -grad\Phi = -\nabla\Phi \tag{1}$$

at any point of interest. For fields without space charge the field region surrounding charges obeys

$$div\vec{D} = 0. \tag{2}$$

Combining the constitutive equation $\vec{D} = \varepsilon\vec{E}$ and the relation 1 the *Laplace's equation* is obtained:

$$div\, grad\Phi = \triangle\Phi = 0 \tag{3}$$

For fields with space charge the *Poisson equation* can be defined:

$$\triangle\Phi = -\frac{\rho}{\varepsilon} \tag{4}$$

The aim is to design and develop components which are free of (partial) discharging processes and the resulting electric fields are lower than the critical breakdown field strength.

in the early days of high voltage energy transmission it was not possible to use computers at all, therefore engineers had to solve the Laplace's equation analytically. There were two differents methods like conformal mapping and solving the differential equation with the method of separation of the variables [3], [4]. Even first methods for optimization of high voltage equipment have been developed by Spielrein [5].

Since the beginning of 1960s numerical methods for the calculation of electrostatic fields have been developed. The most important methods for calculating electrostatic high voltage fields which are implemented in commercial and open source programs will now be shown in detail.

A. *Finite Element Method (FEM)*

is a widely spread numerical method to obtain solutions to the differential equations that approximately describe a wide variety of physical problems like solid, fluid and mechanics, electromagnetism or dynamics [6], [7].

The basic idea of the FEM in high voltage fields is not to solve the Laplace's equation, but is based on the variational calculation, where the solution of field problems is reduced to obtaining the stationary state of the appropriate energy functional. Among enough small parts (finite elements) of the regarded area the field function can be expressed as an analytical function of the position within the finite element yielding a system of algebraic equations. The matrix is of relatively high order with a very sparse coefficient matrix. For solving the equation system iterative or direct methods can be applied.

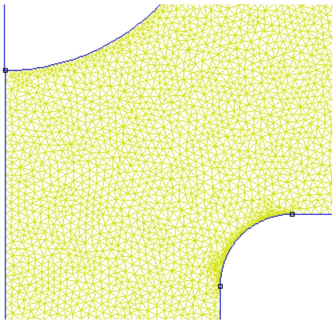

Fig. 1. FEM - Discretization with triangles

For the FEM the entire space of interest must be divided into elements (fig. 1). This permits the calculation of non-linear properties of the space but the discretization results in a very high number of elements [8]. Furthermore a closed field space is required as open fields cannot be calculated directly.

B. Charge Simulation Method (CSM)

is based on the simple principle that the real surface charges on electrodes or dielectric interfaces are replaced by a suitable set of simulation charges (point, line and ring charges) placed in vacuum [9].

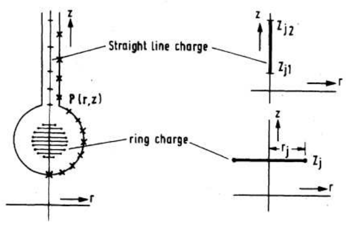

Fig. 2. CSM - Discretization with charges

The position and type of the charges are predetermined, their magnitude is unknown. These magnitudes of the simulation chargse have to be calculated such that their integrated effect satisfies the boundary conditions exactly at a selected number of collocation points.

The superposition of the potentials of the charges must be equal to the boundary condition Φ_B of the collocation point B:

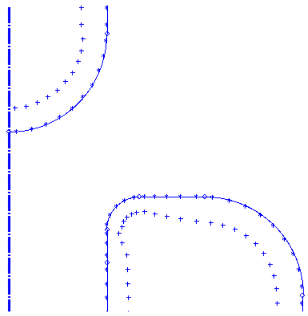

Fig. 3. Position of the charges

$$\Phi = \sum_{j=1}^{n} p_j \cdot Q_j$$

where Q_j is the magnitude of the j-th charge and p_j is its assigned potential coefficient. Assembling all charges we get a system of n linear equations for the n charges:

$$[P][Q] = \{\Phi_B\}$$

To calculate arrangements with more than two dielectrics the CSM has been improved by the *region-oriented* charge simulation method [10]. An important advantage of this method is its ability to solve problems with thin conducting foils and thin dielectric layers.

The CSM was extended by the introduction of equivalent area charges placed directly on the surface of the boundaries. This procedure is called *Surface Charge Simulation Method* [11].

C. Boundary Element Method (BEM)

In the *Boundary Element Method*, the integral equations are translated into a system of algebraic equations replacing the integral by sums of well selected surfaces across the boundaries between two media with different characteristics [12]. Across the boundary element the field is expressed as an analytical function evaluated at certain interpolation nodes. The coefficient matrix is of relatively low order, but it is a full matrix.

Equation 5 represents a Fredholm integral equation of first kind:

$$\phi(I) = \frac{1}{4\pi\epsilon_0}\left[\sum_{p=1}^{P}\int_{S_p}\frac{\sigma(M)}{r_{MI}}dS_p + \sum_{d=1}^{D}\int_{S_d}\frac{\sigma(N)}{r_{NI}}dS_d\right] \quad (5)$$

It can also be applied for capacitive-resistive field calculation if the potential and charges are taken as complex quantities instead of having real values only [8].

The Boundary Element Method can deal with arbitrarily complex geometries and can be perfectly used for 3D optimization since only the surfaces need to be discretized in spite of the entire field space when using FEM.

III. Application Example

The application example for the investigations is a standard 20 kV disconnector. Figure 4 shows the dimensions of the setup.

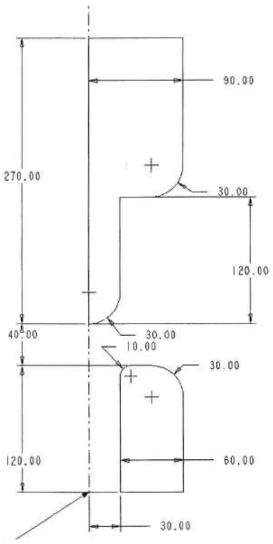

Fig. 4. 2D sketch of the application example

Figure 5 shows a 3D model of the disconnector. The high voltage conductor is located on the upper side. The lower electrode is grounded. For FEM a closed field space is necessary, therefore an additional surrounding boundary is implemented in the FEM simulation model.

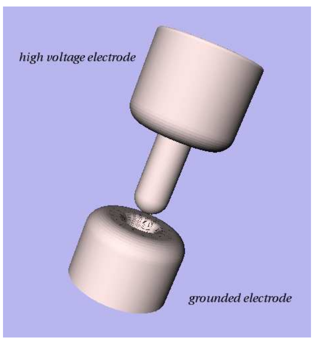

Fig. 5. 3D model of the disconnector

IV. Calculations and Results

The calculations are performed with three different numerical simulation programs: For FEM a tool called *FEMM* is used, which is able to calculate 2D and 3D-axisymmetric geometries [13]. The simulation software for the CSM called *xelfi* and BEM called *Xtwin* were developed at the Chair of High Voltage Engineering and Switchgear Technology in Munich [14] and can also handle 2D and 3D-axisymmetric problems.

To show the results six points in the field space are selected (see fig. 6). The calculations are made with a normalized voltage of $U = 1kV$. The results are presented in table I.

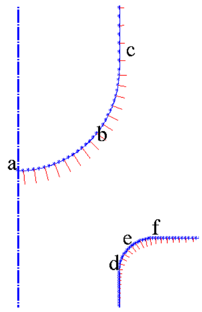

Fig. 6. Investigated areas

As shown in table I, the results are nearly the same, however, when using FEM it is necessary to create a very fine mesh in order to obtain reasonable results at all points.

	E in V/mm		
point	BEM	CSM	FEM
a	57.32	57.44	56.28
b	45.02	45.24	43.82
c	28.23	28.27	25.46
d	11.82	11.61	11.45
e	28.75	28.53	29.39
f	26.78	26.81	26.88

TABLE I
Results of the evaluation

	Number of elements		
	BEM	CSM	FEM
Σ	188	200	265.325

TABLE II
Number of elements in different solution methods

According to the fact that FEM must discretize the entire field space, the number of elements which are necessary to define the problem is tremendously high in contrast to CSM and BEM. Table II underlines this.

With the so called integral methods (CSM and BEM) only electrodes have to be taken into account, and one can handle the entire problem with only a fraction of the elements (200 as opposed 265.325 elements for FEM).

Therefore it is highly recommended to use integral methods for calculating electrostatic fields. For the calculation and optimization of real 3D fields, modern BEM tools appear to be the only reasonable solution.

V. Optimization

20 years ago first investigations were made at the Chair of High Voltage Engineering and Switchgear Technology of TUM in order to use automatic computer-based optimization algorithms [15]. Today modern computer techniques allow the calculation of arbitrarily complex 3D geometries in order to improve the field distribution. For this, BEM is used within a complex toolchain including mathematical optimization procedures.

First of all it is necessary to create a so called parametric model of your problem so that CAD software can automatically (re-)generate the simulation model. With this set of parameters (which describe the model) it is possible to apply mathematical optimization algorithms which can minimize the objective function. In case of electrostatic fields this function is the maximum of the electric field depending on the set of geometry parameters.

Different mathematical optimization methods have been investigated for practical use in numerical field calculation programs [15]. More recent investigations on this topic use a fast Kriging-based strategy for the optimization of electrical devices [16].

With the genetic mathematical algorithm CMA-ES[17], a standard switchgear which design is already close to an optimum, can be further improved.

Figure 7 shows the geometry of the application example, a real world three phase switchgear with the task to optimize the shielding electrodes.

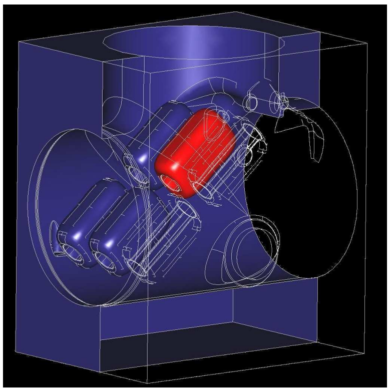

Fig. 7. Switchgear to be investigated.

For the calculations one electrode is applied with 1050 volts. The other five electrodes are grounded.

The initial maximum field at the shield electrode amounts to 35.2 V/m. Note that the field distribution is very inhomogeneous.

For the automatic optimization a parametric model with 15 different geometry parameters is defined. The parameters can be obtained by cuts through all three axes (see fig. 9).

It is necessary to define upper and lower bounds for these parameters in order to obtain feasible geometries.

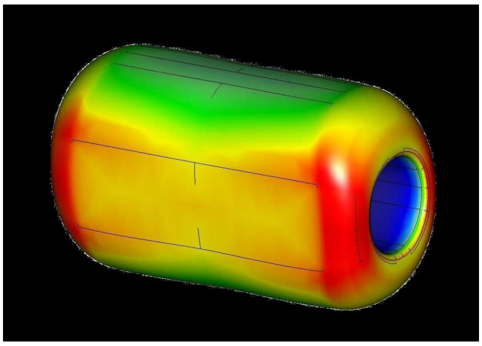

Fig. 8. Shield electrode to be optimized.

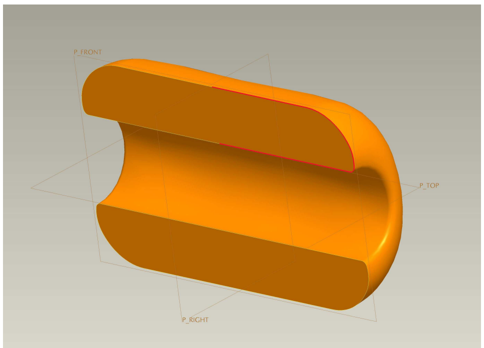

Fig. 9. Cut to obtain set of parameters

A. *Minimizing Runtimes*

For the calculation it is useful to operate with massive parallel simulation methods. BEM field calculation is a very suitable application for parallel computing. Every single step of the calculation process (creating the coefficient matrix, solving matrix and post calculations to obtain potentials and field strength at all contour points) can be implemented in parallel. Furthermore, the optimization itself can also be run in parallel. Figure 11 shows the development of an optimization process.

The calculation time on an average high performance workstation cluster[1] was about 116 hours to obtain an optimum of field reduction.

Number of iterations	260
Number of evaluations	3900
Number of analysis	3012
Calculation time	116.12 hours
Initial value	35.2 V/m
Final optimized value	31.2 V/m
Field reduction	11.34 %

TABLE III
Parameter of the optimization

[1] in 2010

Feature	Dimension name	Value	Lower bound	Upper bound
curve – feature 7	d128	84.24	80.00	95.00
curve – feature 7	d131	45.00	35.00	500.00
curve – feature 7	d133	5.00	1.00	7.50
curve – feature 7	d134	5.47	2.00	8.00
curve – feature 7	d135	30.00	10.00	35.00
curve – feature 8	d139	5.00	1.00	7.50
curve – feature 8	d140	5.20	2.00	8.00
curve – feature 8	d141	35.00	35.00	500.00
curve – feature 8	d143	25.00	20.00	30.00
curve – feature 8	d144	37.36	37.00	42.00
curve – feature 8	d145	15.00	10.00	20.00
curve – feature 8	d150	64.00	64.00	72.00
curve – feature 16	d147	308.34	100.00	2000.00
curve – feature 16	d148	91.00	65.00	1000.00
curve – feature 16	d149	55.00	1.00	60.00

Fig. 10. List of parameters

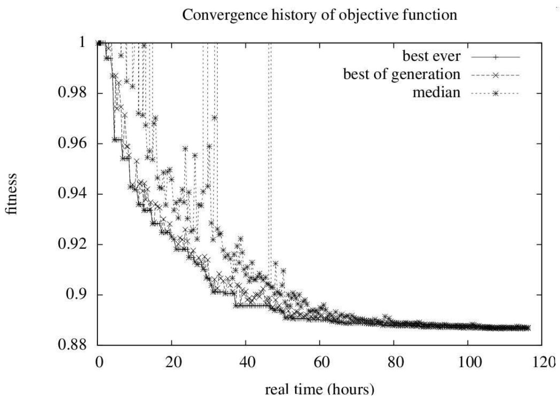

Fig. 11. Optimization process

Finally a new electrode shape was found by the algorithm which yields a kind of saddle on top of the shielding electrode. This change in geometry forces a reduction of field strength of about 11.34 % which is an excellent result. Figure 12 shows the optimized shape of the shielding electrode. The field distribution is more homogeneous, and the maximum field is reduced to 31.2 V/m.

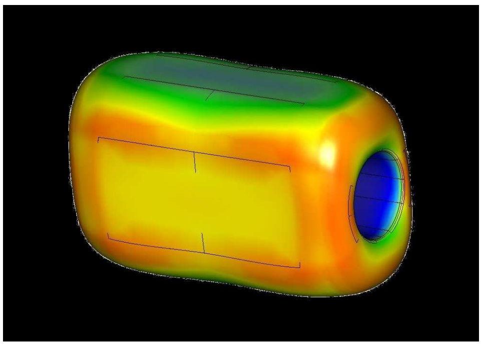

Fig. 12. Optimized result

VI. Conclusions

Numerical methods like FEM, CSM and BEM, which are able to calculate electrostatic fields, have been discussed. For 3D calculation and especially for automatic optimization the Boundary Element Method is the best numerical simulation method, due to the fact that the dimensions of the underlying equation system are significantly lower than for FEM. New mathematical procedures for automatic optimization have been successfully implemented and tested.

Authors

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